She reasoned as follows:

$$\begin{aligned}\sin^2\alpha - \sin^2\beta &= (\sin\alpha + \sin\beta)(\sin\alpha - \sin\beta)\\ &= \sin(\alpha+\beta)\sin(\alpha-\beta)\,.\end{aligned}$$

What criticism do you have of her reasoning?

7. Check the identity in Problem 6 using $\alpha = 30°$, $\beta = 60°$, on your calculator. You will find that, despite Phoebe's specious reasoning, the identity is true for these values. Is this a coincidence?

8. Prove that $\sin^2\alpha - \sin^2\beta = \sin(\alpha+\beta)\sin(\alpha-\beta)$.

9. Prove that $\cos^2\beta - \cos^2\alpha = \sin(\alpha+\beta)\sin(\alpha-\beta)$.

10. Without using your calculator, find the numerical value of $\sin 18° \cos 12° + \cos 18° \sin 12°$.

11. a) Without using your calculator, try to find the numerical value of $\sin 113° \cos 307° + \cos 113° \sin 307°$.
    b) Now use your calculator to check the result.
    c) Did you use the addition formulas in part (a)? Remember that we have proved the addition formulas only for positive acute angles. Doesn't it look like they work for larger angles as well?

12. Simplify the expression $\sin 2\alpha \cos\alpha - \cos 2\alpha \sin\alpha$.

13. Simplify the expression $\sin(\alpha+\beta)\sin\beta + \cos(\alpha+\beta)\cos\beta$.

14. Simplify the expression

$$\frac{\sin(\alpha+\beta) - \cos\alpha\sin\beta}{\cos(\alpha+\beta) + \sin\alpha\sin\beta}\,.$$

15. For any angle $\alpha < \pi/4$, show that

$$\sin\left(\alpha + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\sin\alpha + \cos\alpha)\,.$$

16. For any acute angles α and β for which $\cos\alpha\cos\beta \neq 0$, show that

$$\frac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta} = 1 - \tan\alpha\tan\beta\,.$$

17. Use the law of cosines and the figure drawn for the second beautiful proof to give a direct derivation of the formula for $\cos(\alpha+\beta)$.